Wormhole Geometries In $f(R,T)$ Gravity

Tahereh Azizi*
Department of Physics, Faculty of Basic Sciences,
University of Mazandaran,
P. O. Box 47416-95447, Babolsar, IRAN

We study wormhole solutions in the framework of $f(R,T)$ gravity where R is the scalar curvature, and T is the trace of the stress-energy tensor of the matter. We show that in this modified gravity scenario, the matter threading the wormhole may satisfy the energy conditions, so it is the effective stress-energy that is responsible for violation of the null energy condition.

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Key Words: Modified Gravity, Traversable Wormhole, Energy Conditions

I. INTRODUCTION

One of the great discoveries in modern cosmology has been the accelerating expansion of the universe[1–3]. Among the various interesting possibilities invoked in order to explain the cosmic speed up, $f(R)$ modified gravity models (R is the scalar curvature) have attracted a lot of attention(see [4–11] and references therein). The Einstein field equation of General Relativity was first derived from an action principle by Hilbert, by adopting a linear function of the scalar curvature, R, in the gravitational Lagrangian density. An extension of the Einstein-Hilbert Lagrangian to the $f(R)$ gravity scenario can be viewed natural as there is no a priori reason why the gravitational action should be linear in the Ricci scalar R. Furthermore, higher order terms can naturally appear in low energy effective Lagrangians of quantum gravity and string theory.

In ref [12], a generalization of $f(R)$ modified theories of gravity was proposed by including in the theory a coupling of an arbitrary function of the Ricci scalar with the trace of the stress-energy tensor T , i.e. $f(R,T)$ gravity. They investigate some astrophysical and cosmological aspects of the scenario by choosing the several functional forms of f (see also [13]).

In this paper, we explore the possibility that static and spherically symmetric traversable wormhole geometries are supported by $f(R,T)$ gravity. We show that, in this modified theory, static wormhole throats respecting the null energy condition (NEC) can exist. Note that as is widely known, traversable wormholes as solutions to the Einstein equations can only exist with exotic matter which violates the null energy condition [14–16]. The null energy condition holds if $T_{\mu\nu}n^\mu n^\nu \geq 0$ for any null vector field n^μ. The search of realistic physical models providing the wormhole existence represents an important direction in wormhole physics. Various models of such kind include scalar fields [17–19]; wormhole solutions in semi-classical gravity [20, 21]; solutions in modified gravity with [22] and without [23, 24] nonminimal curvature-matter coupling; wormholes on the brane [25, 26]; wormholes supported by generalized Chaplygin gas [27]; wormhole solutions in Einstein-Gauss-Bonnet theory [28]; modified teleparallel gravity [29], etc (for instance see [30] and references therein).

This paper is organized as follows. In section II, we present a brief review of the fundamental concepts of $f(R,T)$ gravity, the action of the scenario and equations of motion. We explore the wormhole geometries in $f(R,T)$ gravity in section III. Firstly, we introduce the space-time metric and the necessary conditions to have a traversable wormhole solution. Secondly, we obtain solutions of the field equations in two special functional form of f. In each cases we specific an equation of state for the matter field and investigate the energy conditions violation. We impose that the matter threading the wormhole satisfies the energy conditions. So, it is the effective stress-energy tensor that responsible for the violation of the null energy condition. Finally, our summery and conclusions are presented in section IV.

II. $f(R,T)$ GRAVITY

The action of $f(R,T)$ gravity is of the form [12]

$$\mathcal{S} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\, f\left(R,T\right) + \int d^4x\sqrt{-g}\,\mathcal{L}_m \qquad (1)$$

Here $f\left(R,T\right)$ is an arbitrary function of the scalar curvature, $R = R^\mu_\mu$, and the trace $T = T^\mu_\mu$ of the stress-energy tensor of the matter, $T_{\mu\nu}$. $\mathcal{L}_m$ is the Lagrangian density of the matter and is related to the stress-energy tensor as follows [12]

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta g^{\mu\nu}}\,. \qquad (2)$$

Assuming that the Lagrangian density of matter $\mathcal{L}_m$ depends only on the metric $g_{\mu\nu}$, we deduce that

$$T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - 2\frac{\partial\mathcal{L}_m}{\partial g^{\mu\nu}}\,. \qquad (3)$$

* t.azizi@umz.ac.ir

Varying the action (1) with respect to the metric provides the field equation of $f(R,T)$ gravity [12]

$$f_R\left(R,T\right)\left(R_{\mu\nu}-\frac{1}{3}Rg_{\mu\nu}\right)+\frac{1}{6}f\left(R,T\right)g_{\mu\nu}=$$

$$8\pi\left(T_{\mu\nu}-\frac{1}{3}Tg_{\mu\nu}\right)-f_T\left(R,T\right)\left(T_{\mu\nu}-\frac{1}{3}Tg_{\mu\nu}\right)$$

$$-f_T\left(R,T\right)\left(\Theta_{\mu\nu}-\frac{1}{3}\Theta g_{\mu\nu}\right)+\nabla_\mu\nabla_\nu f_R\left(R,T\right). \quad (4)$$

where we have denoted $f_R\left(R,T\right)=\partial f\left(R,T\right)/\partial R$ and $f_T\left(R,T\right)=\partial f\left(R,T\right)/\partial T$, respectively and

$$\Theta_{\mu\nu}\equiv g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}. \quad (5)$$

This term also can be obtained as [12]

$$\Theta_{\mu\nu}=-2T_{\mu\nu}+g_{\mu\nu}\mathcal{L}_m-2g^{\alpha\beta}\frac{\partial^2\mathcal{L}_m}{\partial g^{\mu\nu}\partial g^{\alpha\beta}}. \quad (6)$$

In this paper, we assume that the stress-energy tensor of the matter is given by the perfect fluid form such as

$$T_{\mu\nu}=\left(\rho+p\right)u_\mu u_\nu-pg_{\mu\nu}, \quad (7)$$

where ρ is the energy density, p is the pressure and u^μ describes the four-velocity. We also assume that the matter Lagrangian takes the form $\mathcal{L}_m=-\rho$ (see [23, 31, 32]). Thus, with these assumption, Eq. (5) takes the form

$$\Theta_{\mu\nu}=-2T_{\mu\nu}-\rho g_{\mu\nu}. \quad (8)$$

III. WHORMHOLE GEOMETRIES IN $f(R,T)$ GRAVITY

A. Spacetime metric

We consider the wormhole metric is given by the following line element

$$ds^2=-e^{2\Phi(r)}dt^2+\frac{dr^2}{1-b(r)/r}+r^2\left(d\theta^2+\sin^2\theta\,d\phi^2\right), \quad (9)$$

where $\Phi(r)$ and $b(r)$ are arbitrary functions of the radial coordinate, r, denoted as the redshift function, and the shape function, respectively [14]. The radial coordinate r is non-monotonic such that it decreases from infinity to a minimum value r_0, representing the location of the throat of the wormhole, where $b(r_0)=r_0$, and then it increases from r_0 back to infinity. To have a traversable wormhole solution, it is necessary to impose the flaring out condition, given by $(b-b'r)/b^2>0$, [14, 22], and at the throat $b(r_0)=r=r_0$, the conditions $b'(r_0)<1$ and $1-b(r)/r>0$ are imposed. For the wormhole to be traversable, one must demand that there are no horizons present, which are identified as the surfaces with $e^{2\Phi}\to 0$, so that $\Phi(r)$ must be finite everywhere. In the following analysis, for simplicity, we consider that the redshift function is constant so, $\Phi'=0$.

B. Special Case: $f\left(R,T\right)=R+2f(T)$

1. Gravitational field equations

In this section, we assume that $f\left(R,T\right)=R+2f(T)$, where $f(T)$ is an arbitrary function of the trace of the stress-energy tensor. The gravitational field equations (4), by the definition (8) take the form

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=8\pi T_{\mu\nu}+2FT_{\mu\nu}+\left(2\rho F+f\right)g_{\mu\nu}. \quad (10)$$

where $f=f(T)$ and $F=\frac{df}{dT}$. Supposing $8\pi\equiv 1$, this equation can be recast in the form

$$G_{\mu\nu}\equiv R_{\mu\nu}-\frac{1}{2}R\,g_{\mu\nu}=T^{(\mathrm{eff})}_{\mu\nu}, \quad (11)$$

where the effective stress-energy tensor is defined by $T^{(\mathrm{eff})}_{\mu\nu}=T^{(\mathrm{m})}_{\mu\nu}+\tilde{T}^{(\mathrm{m})}_{\mu\nu}$. The latter term is given by

$$\tilde{T}^{(\mathrm{m})}_{\mu\nu}=2FT_{\mu\nu}+\left(2\rho F+f\right)g_{\mu\nu}$$

For the matter content of the wormhole, we consider an anisotropic fluid source whose stress-energy tensor is satisfied the energy conditions and is given by [14]

$$T_{\mu\nu}=(\rho+p_t)u_\mu\,u_\nu+p_t\,g_{\mu\nu}+(p_r-p_t)\chi_\mu\chi_\nu, \quad (12)$$

Here ρ, p_t and p_r are the energy density, the perpendicular (to the inhomogeneous direction) pressure, and the parallel pressure respectively as measured in the fluid elements rest frame. The vector u_μ is the fluid four-velocity and χ_μ is a space-like vector orthogonal to u_μ. With these considerations, the stress-energy tensor takes a diagonal form, i.e., $T^\mu{}_\nu=\mathrm{diag}[-\rho(r),p_r(r),p_t(r),p_t(r)]$. Thus, the gravitational field equations (11) are given as follows

$$\frac{b'}{r^2}=\rho-f, \quad (13)$$

$$-\frac{b}{r^3}=p_r\left(1+2F\right)+2\rho F+f, \quad (14)$$

$$\frac{b-b'r}{2r^3}=p_t\left(1+2F\right)+2\rho F+f. \quad (15)$$

Now we assume that $f(T)=\lambda T$, where λ is a constant [12] and with Eq. (12), $T=-\rho+p_r+2p_t$. Thus the field equations (13)-(15) yield the following relationships:

$$\rho=\frac{b'}{r^2(1+2\lambda)}, \quad (16)$$

$$p_r=-\frac{b}{r^3(1+2\lambda)}, \quad (17)$$

$$p_t=\frac{(b-b'r)}{2r^3(1+2\lambda)}. \quad (18)$$

These equations describe the matter threading the wormhole, as a function of the shape function and the coupling parameter λ. Note that in the case $\lambda=0$, the general

relativistic limit can be recovered. The system of equations (16)- (18) are three equations with four unknown functions $\rho(r)$, $p_r(r)$, $p_t(r)$ and $b(r)$. There are different strategies to solve the field equations. For example, by specifying an equation of state for the matter field, one can obtain the shape function and the stress-energy components.

2. *Energy conditions*

As was already mentioned in the introduction, the existence of the wormhole solution in general relativity relies on the violation of the null energy condition. The null energy condition holds if

$$T_{\mu\nu}n^{\mu}n^{\nu} \geq 0$$

for any null vector field n^{μ}. However, if the theory of gravity is chosen to be more complicated than Einstein gravity, one may circumvent this issue and possess a throat region which respects energy conditions. Thus, considering a radial null vector, the violation of the NEC, i.e., $T^{(\text{eff})}_{\mu\nu}n^{\mu}n^{\nu} < 0$ takes the following form

$$\rho^{(\text{eff})} + p_r^{(\text{eff})} = (1+2\lambda)(\rho + p_r) < 0. \qquad (19)$$

On the other hand, with the field equations (16)-(18) we deduce that

$$\rho^{(\text{eff})} + p_r^{(\text{eff})} = \frac{b'r - b}{r^3}. \qquad (20)$$

Using the flaring out condition i.e., $(b'r - b)/b^2 < 0$, this term is negative. If we consider the matter threading the wormhole satisfies the energy conditions, imposing the weak energy condition (WEC), given by $\rho \geq 0$ and $\rho + p_r \geq 0$, so the coupling parameter λ is limited to $\lambda \leq \frac{-1}{2}$.

3. *Special solution: $p_r = \alpha\rho$*

In this section, we adopt a special equation of state for the matter field threading the wormhole and obtain the solution of the field equations (16)-(18). An interesting equation of state is a linear relation between the radial pressure and the energy density, i.e. $p_r = \alpha\rho$, where α is a constant. Using this equation of state provides the following shape function

$$b(r) = r_0\left(\frac{r_0}{r}\right)^{1/\alpha}. \qquad (21)$$

Note that with the condition at the throat $1 - \frac{b(r)}{r} \geq 0$ (which is equivalent to $r - b(r) \geq 0$), the allowed region for the parameter α is restricted to $\alpha > 0$ and $\alpha < -1$. This issue is depicted in Fig. 1. In Fig. 2 and Fig. 3, we have plotted the shape function versus r for $\alpha = 0.6$ and $\alpha = -1.5$ respectively. As the figures show, the fundamental wormhole condition, i.e. $b(r) < r$ is fulfilled.

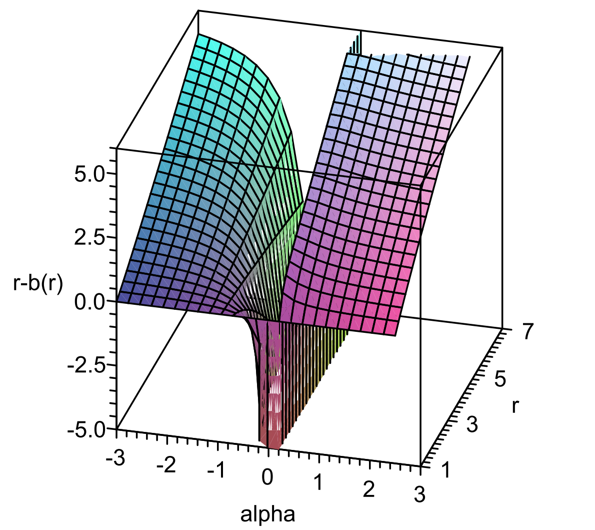

FIG. 1. The shape function restriction $r - b(r) > 0$ is plotted for the value $r_0 = 1$.

Using the shape function (21) with gravitational field equations (16)-(18), the stress-energy components are given by

$$p_r = \alpha\rho = -\frac{Cr^{-(3+1/\alpha)}}{(1+2\lambda)}, \qquad (22)$$

and

$$p_t = \frac{C(\alpha+1)r^{-(3+1/\alpha)}}{2\alpha(1+2\lambda)}, \qquad (23)$$

where $C = r_0^{1+1/\alpha}$. Fig. 4 shows the energy density versus r for $\alpha = 0.6$ and $\alpha = -1.5$. Clearly, choosing a negative value for α in the allowed region, leads to a negative energy density for the matter threading the wormhole.

Now, using the expression (22), the NEC condition is depicted in Fig. 5 for $\alpha = 0.6$ and $\alpha = -1.5$ respectively. As the figures show, the stress-energy tensor satisfies the null energy condition. As a result, for a positive value of α, the weak energy condition is satisfied. On the other hand, by choosing a negative value for α in the allowed region, only the null energy condition is satisfied.

C. Case: $f(R,T) = f_1(R) + f_2(T)$

As a second example, we consider the case in which the function f is given by $f(R,T) = f_1(R) + f_2(T)$, where

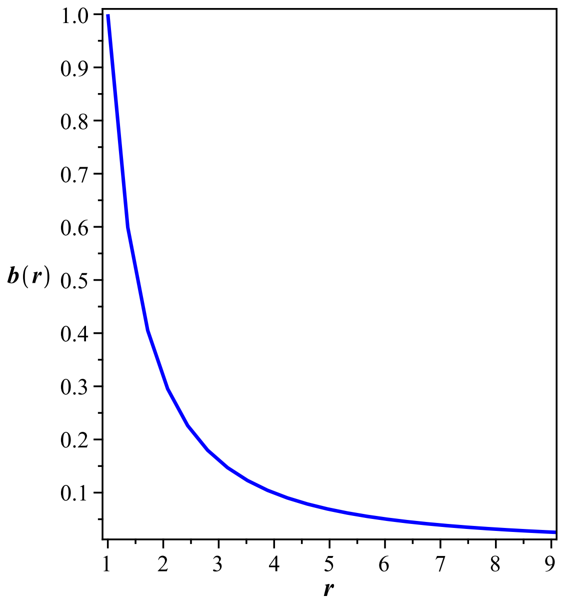

FIG. 2. The shape function $b(r)$ versus r for $\alpha = 0.6$ and$r_0 = 1$.

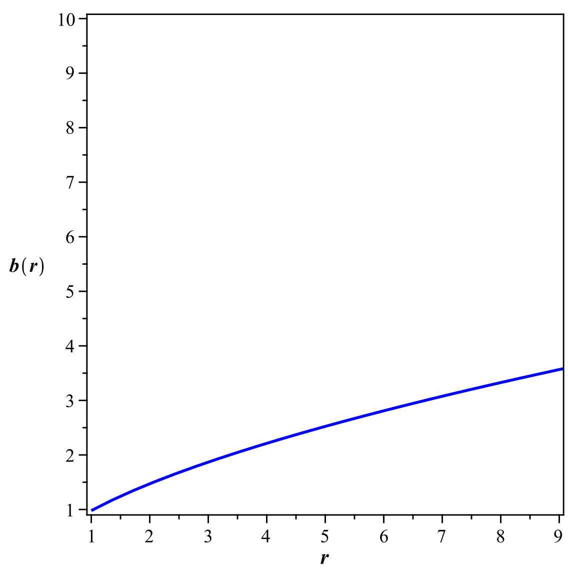

FIG. 3. The behavior of $b(r)$ versus r for $\alpha = -1.5$ and $r_0 = 1$.

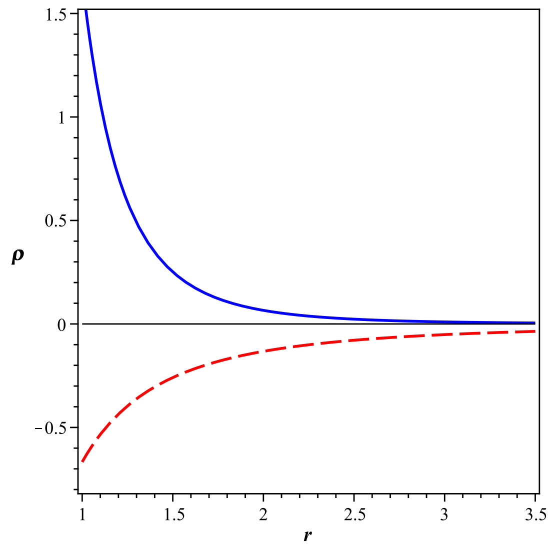

FIG. 4. The energy density versus r for $\alpha = 0.6$ (solid line) and $\alpha = -1.5$ (dashed line) with $\lambda = -1$ and $r_0 = 1$.

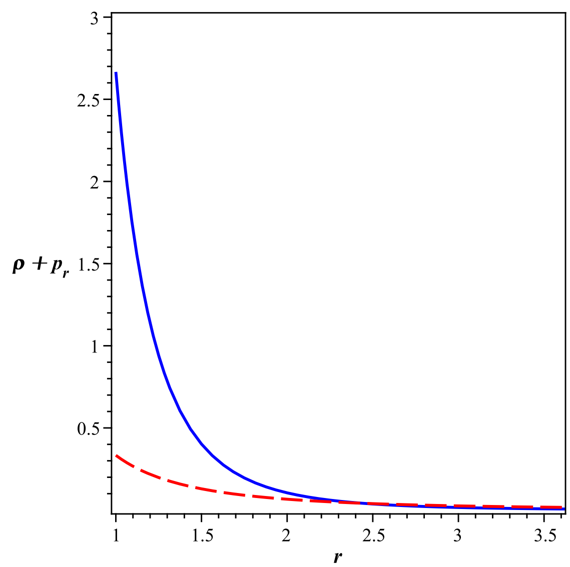

FIG. 5. The null energy density is satisfied for $\alpha = 0.6$ (solid line) and $\alpha = -1.5$ (dashed line) with $\lambda = -1$ and $r_0 = 1$.

$f_1(R)$ and $f_2(T)$ are arbitrary functions of the scalar curvature and the trace of the stress-energy tensor, respectively. For an arbitrary matter source the gravitational field equations are given by [12]

$$f_1'(R)R_{\mu\nu} - \frac{1}{2}f_1(R)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_1'(R) = 8\pi T_{\mu\nu} + f_2'(T)T_{\mu\nu} + \left[f_2'(T)p + \frac{1}{2}f_2(T)\right]g_{\mu\nu} \,. \tag{24}$$

where the prime denotes a derivative with respect to the argument. Assuming $f_2(T) = \lambda T$, leads to $f_2'(T) = \lambda$, so

the above equation can be recast as follows

$$f_1'(R)R_{\mu\nu} - \frac{1}{2}f_1(R)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_1'(R) = 8\pi(1+\lambda)T_{\mu\nu} + \lambda(\rho + \frac{1}{2}T)g_{\mu\nu}\,. \tag{25}$$

Taking the trace of this equation gives

$$f_1(R) = \frac{1}{2}\left(FR + 3\Box F - \widetilde{T}\right) \tag{26}$$

where $F = f_1'(R)$ and $\widetilde{T} = (1+3\lambda)\,T + 4\rho\lambda$. With the above definitions, the field equation (25) becomes

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T^{(eff)}_{\mu\nu} \tag{27}$$

where

$$T^{(eff)}_{\mu\nu} = \frac{1}{F}\left[(1+\lambda)\,T_{\mu\nu} + \lambda\left(\rho + \frac{1}{2}T\right)g_{\mu\nu}\right] + T^{(curve)}_{\mu\nu} \tag{28}$$

$T^{(curve)}_{\mu\nu}$ is associated to the curvature stress-energy tensor defined by

$$T^{(curve)}_{\mu\nu} = \frac{1}{F}\left[\nabla_\mu\nabla_\nu F - \frac{1}{4}g_{\mu\nu}\left(RF + \Box F + \widetilde{T}\right)\right] \tag{29}$$

The equation (27) with the anisotropic stress-energy (12) provides the gravitational field equations in the following form

$$\frac{b'}{r^2} = \frac{1}{F}\left[\left(1+\frac{1}{2}\lambda\right)\rho - \frac{1}{2}\lambda\,(p_r + 2p_t)\right] + \frac{I}{F}, \tag{30}$$

$$-\frac{b}{r^3} = \frac{1}{F}\left[\left(1+\frac{3}{2}\lambda\right)p_r + \frac{1}{2}\lambda\,(\rho + 2p_t)\right] + \frac{J}{F}, \tag{31}$$

$$\frac{b-b'r}{2r^3} = \frac{1}{F}\left[(1+2\lambda)\,p_t + \frac{1}{2}\lambda\,(\rho + p_r)\right] + \frac{K}{F} \tag{32}$$

where I, J and K are functions of the radial coordinate r and defined by

$$I(r) = \frac{1}{4}\,(FR + \Box F + T)\;,$$
$$J(r) = (1-\frac{b}{r})\left(F'' + F'\frac{b-b'r}{2r^2(1-\frac{b}{r})}\right)\;,$$
$$K(r) = (1-\frac{b}{r})\frac{F'}{r}\,. \tag{33}$$

In these expressions, the prime denotes a derivative with respect to the r. The Ricci scalar R is given by

$$R = \frac{2b'}{r^2} \tag{34}$$

and $\Box F$ can be obtained as

$$\Box F = \left(1-\frac{b}{r}\right)\left[F'' - \frac{b'r-b}{2r^2(1-b/r)}\,F' + \frac{2F'}{r}\right]\,. \tag{35}$$

from the gravitational field equations (30)-(32), the matter components threading the wormhole can be deduced in the following relationships

$$\rho = \frac{b'F}{r^2(1+\lambda)} - \frac{\lambda}{2(1+\lambda)(1+2\lambda)}\Box F\,, \tag{36}$$

$$p_r = -\frac{bF}{r^3(1+\lambda)} - \frac{g}{(1+\lambda)} + \frac{\lambda}{2(1+\lambda)(1+2\lambda)}\Box F\,, \tag{37}$$

$$p_t = \frac{(b-b'r)F}{2r^3(1+\lambda)} - \frac{h}{(1+\lambda)} + \frac{\lambda}{2(1+\lambda)(1+2\lambda)}\Box F\,. \tag{38}$$

The system of these three equations consist of 5 undefined functions ,i.e: $b(r)\,,\rho(r)\,,p_r(r)\,,p_t(r)$ and $F(r)$. So one can adopt several strategies to find a solution of the field equations. To have a solution, one approach is choosing a viable shape function and use a specific equation of state for the matter components. In the following, we adopt a solution of shape function such as $b(r) = \frac{r_0^2}{r}$ [22] and specify two forms of equation of state for the matter field to investigate the energy conditions. Note that the violation of the null energy condition in this case, takes the form

$$\rho^{(\mathrm{eff})} + p_r^{(\mathrm{eff})} = \frac{(1+2\lambda)\,(\rho + p_r)}{F} + \frac{J(r)}{F} < 0. \tag{39}$$

Using the field equations (36) and (37), this inequality is given by

$$\rho^{(\mathrm{eff})} + p_r^{(\mathrm{eff})} = \frac{b'r - b}{r^3}\,. \tag{40}$$

As mentioned previously, taking into account the flaring out condition, this term is negative. If we consider the matter threading the wormhole satisfying the energy conditions, so the extra terms due to the higher order curvature and the trace of the stress-energy tensor are responsible for the NEC violation.

1. *Case $p_r = \alpha\rho$*

Specifying an equation of state in the form $p_r = \alpha\rho$, leads to the following relation for $F(r)$

$$F(r) = C_1 r^{\left(\frac{-1}{A}\right)}\mathrm{P}^{\gamma}_{\sigma}(\mathrm{z}) + \mathrm{C_2 r}^{\left(\frac{-1}{A}\right)}\mathrm{Q}^{\gamma}_{\sigma}(\mathrm{z}) \tag{41}$$

where $\mathrm{P}^{\gamma}_{\sigma}(\mathrm{z})$ and $\mathrm{Q}^{\gamma}_{\sigma}(\mathrm{z})$ are associated Legendre function of the first and second kind of type 2 respectively and we have defined $\gamma = \frac{1}{A}$, $\sigma = \frac{\sqrt{-A\alpha+A+1}}{A}$ and $z = \sqrt{1-\frac{r^2}{{r_0}^2}}$. C_1 and C_2 are constants of integration and A is defined by $A = \frac{\lambda(3-\alpha)-2}{2(1+2\lambda)}$. The curvature scalar, Eq. (34), is given by $R = -2r_0^2/r^4$ and its inverse provides $r = (-2r_0^2/R)^{1/4}$. At the throat, this quantity is

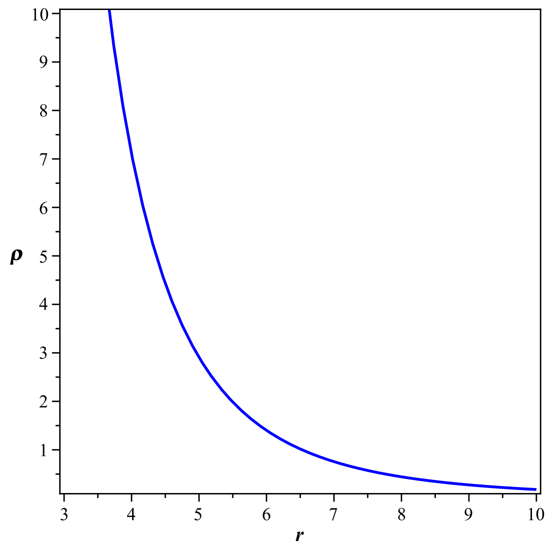

FIG. 6. The energy density versus r for $\alpha = -0.3$, $C_1 = -1$, $C_2 = 1$, $r_0 = 1$ and $\lambda = 1$.

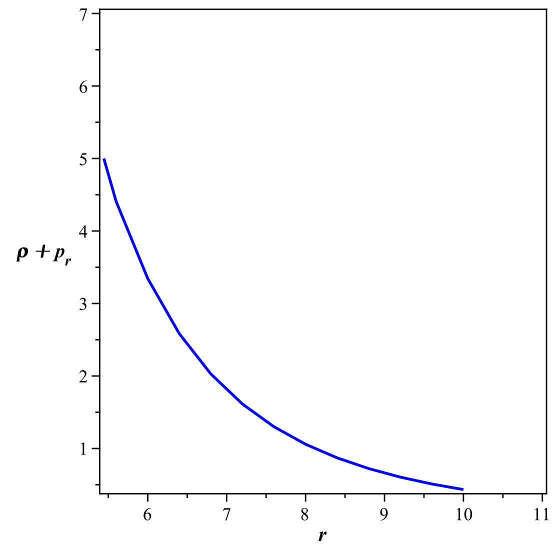

FIG. 7. The stress-energy tensor satisfying the null energy condition for the specific case of the equation of state in the form $p_r = \alpha\rho$. The values of the parameter space are $\alpha = -0.3$, $C_1 = -1$, $C_2 = 1$, $r_0 = 1$ and $\lambda = 1$.

obtained as $r_0 = (-2/R_0)^{1/2}$. Thus the form of $F(R)$ can be written as follows

$$F(R) = C_1 \left(\frac{4}{R_0 R}\right)^{\frac{-1}{4A}} \mathrm{P}^{\gamma}_{\sigma}(\mathrm{v}) + \mathrm{C}_2 \left(\frac{4}{\mathrm{R_0 R}}\right)^{\frac{-1}{4\mathrm{A}}} \mathrm{Q}^{\gamma}_{\sigma}(\mathrm{v}) \tag{42}$$

where $\mathrm{v} = \sqrt{1 - \left(\frac{\mathrm{R_0}}{\mathrm{R}}\right)^{\frac{1}{2}}}$. Equation (42) with equations (36)-(38) leads to the stress-energy components which are too lengthy and we do not write them here explicitly. Instead, we have plotted the energy density and the null energy condition versus r in figures 6 and 7 respectively. We have chosen the values of the parameter space as $\alpha = -0.3$, $C_1 = -1$, $C_2 = 1$, $r_0 = 1$ and $\lambda = 1$. As the figures show, the stress-energy tensor of the matter threading the wormhole satisfies the weak energy condition.

2. *Case $p_r = p_t$*

In this subsection, we take the equation of state in the specific case of an isotropic pressure, i.e. $p_r = p_t$. Thus, the form of $F(r)$ is given by

$$\begin{aligned} F(r) = {} & C_1 HnG\left(-1, \frac{2r_0}{B}, 1 - \frac{3}{B}, 0, 1 - \frac{3}{B}, \frac{1}{2}, -\frac{r}{r_0}\right) \\ & + C_2 HnG\left(-1, \frac{2r_0}{B}, 1, \frac{3}{B}, 1 + \frac{3}{B}, \frac{1}{2}, -\frac{r}{r_0}\right) r^{\frac{3}{B}} \end{aligned} \tag{43}$$

where HnG is the HeunG function ,as a natural generalization of the hypergeometric function, which is the solution of the Heun General equation (see ref. [33]). C_1 and C_2 are constants of integration and B is defined by $B = \frac{2+5\lambda}{2(1+2\lambda)}$. Now, the form of $F(R)$ is obtained as

$$\begin{aligned} F(R) = {} & C_1 HnG\left[-1, \sqrt{\frac{-8}{R_0 B^2}}, 1 - \frac{3}{B}, 0, 1 - \frac{3}{B}, \frac{1}{2}, -(\frac{R_0}{R})^{1/4}\right] \\ & + C_2 HnG\left[-1, \sqrt{\frac{-8}{R_0 B^2}}, 1, \frac{3}{B}, 1 + \frac{3}{B}, \frac{1}{2}, -(\frac{R_0}{R})^{1/4}\right] \left(\frac{R_0}{R}\right)^{\frac{3}{4B}} \end{aligned} \tag{44}$$

Fig. 8 shows the energy density versus r for $\alpha = 0.5$, $C_1 = 1$, $C_2 = -1$, $r_0 = 1$ and $\lambda = 1$. In Fig. 9, the null energy condition is depicted versus r. Consequently, the stress-energy tensor of the matter field satisfies the weak energy condition.

IV. CONCLUSION

The existence of traversable wormholes as solutions to the Einstein equations relies on the presence of some form of exotic matter which violates the null energy condition. However, in the framework of a modified theory of gravity, the situation may be completely different. In this paper, we have investigated wormhole solutions in $f(R,T)$ modified gravity where R is the Ricci scalar and T is the trace of the stress-energy tensor. We have shown that in the context of $f(R,T)$ gravity, traversable wormhole solutions can be obtained, without the need of introducing any form of exotic matter. The violation of the energy conditions, which is essential for the existence of the wormhole solutions [14], is realized via the presence

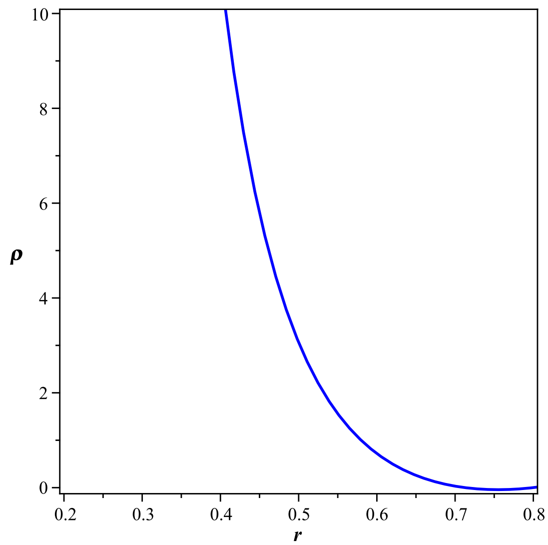

FIG. 8. The energy density versus r for $\alpha = 0.5$, $C_1 = 1$, $C_2 = -1$, $r_0 = 1$ and $\lambda = 1$.

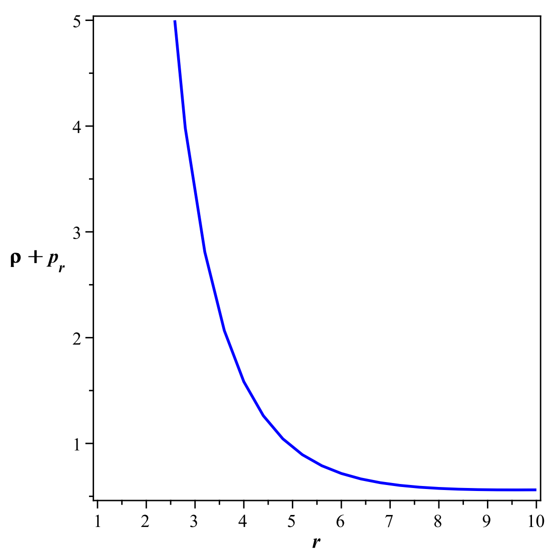

FIG. 9. The stress-energy tensor satisfying the null energy condition for the specific case of the equation of state of an isotropic pressure. The values of the parameter space are the same as Fig. 8

of an effective stress-energy tensor generated by the extra curvature and matter terms.

To find the wormhole solutions in $f(R,T)$ gravity, we have taken several approaches. Firstly, we have assumed that $f(R,T) = R + 2f(T)$, where $f(T)$ is an arbitrary function of the trace of the stress-energy tensor and we have derived the gravitational field equations. Then by specifying an equation of state for the matter threading the wormhole and imposing the flaring out condition at the throat, we have obtained the shape function. We have shown that the stress-energy tensor of the matter threading the wormhole, satisfies the null energy condition. Secendly, we have assumed that $f(R,T) = f_1(R) + f_2(T)$ and obtained the gravitational field equations. To investigate the energy conditions, we have specified a viable shape function and consider two special case for the equation of state for the matter field. We have shown that in each cases, the null and weak energy conditions are fulfilled with a suitable choice of the parameter space of the model.

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